Hardware - Tutorial 2
Boolean Algebra and Karnaugh Maps

Exercice 1: Boolean Algebra

1) Simplify the following expressions:

- $S_1 = (A + B)\cdot(\bar{A} + \bar{B})$
- $S_2 = A\cdot B + \bar{A}\cdot\bar{B} + \bar{A}\cdot B$
- $S_3 = (A + \bar{B})\cdot(A + B) + C\cdot(\bar{A} + B)$
- $S_4 = (A + C + D)\cdot(B + C + D)$
- $S_5 = (A\cdot\bar{B} + A\cdot B + A\cdot C)\cdot(\bar{A}\cdot\bar{B} + A\cdot B + A\cdot\bar{C})$
- $S_6 = (A + \bar{B} + C)\cdot(A + \bar{C})\cdot(\bar{A} + \bar{B})$
- $S_7 = A\cdot B\cdot C + A\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B\cdot\bar{C} + \bar{A}\cdot B\cdot C$
- $S_8 = A\cdot B\cdot C + A\cdot\bar{B}\cdot C + A\cdot B\cdot\bar{C}\cdot D$

2) Calculate and simplify the complement of S1, S5 and S6.

3) Design the NOT, AND and OR gates by using only NAND gantes, then only NOR gates.

Exercice 2: Equality

Demonstrate the following equalities:

- $\overline{A\cdot C + B\cdot\bar{C}} = \bar{A}\cdot C + \bar{B}\cdot\bar{C}$
- $(A + B)\cdot(\bar{A} + C)\cdot(B + C) = (A + B)\cdot(\bar{A} + C)$

Exercice 3: Problem

Given 3 binary variables A, B and C, design an expression S that is true only if the number of true variables is odd.

Exerice 4: 3 variables Karnaugh maps

Given a number N encoded in 3 bits binary (C, B, A) with C the most significant bit, find the most simplified boolean expression of $S = f(N)$ using Karnaugh maps for each of the following:

- $S_1 = 1$ when $N >= 3$
- $S_2 = 1$ when $2 < N <= 6$
- $S_3 = 1$ when $N = 1, 3, 5$
- $S_4 = 1$ when $N = 1, 3, 5$ and S is undefined when $N = 0$ or 7

Exercice 5: Problem

We want to design a circuit that can perform the two's complement of a 3 bits binary number.
This circuit has 3 inputs (C, B, A) and 3 outputs (C', B', A') with C and C' their most significant bits.

1) Write down the truth table of each of theses outputs.
2) Find their most simplified expression using Karnaugh maps.

Exercice 6: 4 variables Karnaugh maps

Given a number N encoded in 4 bits binary (D, C, B, A) with D the most significant bit, find the most simplified boolean expression of $S = f(N)$ using Karnaugh maps for each of the following:

- $S = 1$ when $N >= 10$
- $S = 1$ when $N = 0, 4, 8, 10, 12$ or 14
- $S = 1$ when $N = 0, 2, 5, 7, 8, 10, 13$ or 15
- $S = 1$ when $N = 2, 10, 11$ or 14
- $S = 1$ when $N = 2, 10, 11$ or 14 and S is undefined when $N = 6, 9, 13$ or 15